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VORTEX-FREE PROPULSION IN AN IDEAL FLUID

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A formula for the velocity of a sphere is derived. The sphere is propelled in an ideal incompressible fluid from a state of rest by the fixed normal component of the velocity of the fluid at the permeable surface of the sphere. The fluid flow is a potential flow.

Within the framework of potential flows of an ideal incompressible fluid propulsion (self-propulsion) of bodies from a state of rest is possible owing to a periodic change in shape even though there is no propulsion force [1, 2]. V. L. Sennitskiĭ [3] and V. V. Pukhnachev [4] studied propulsion in a viscous fluid due to the fixed velocity of the fluid on the surface of the body, which was assumed to be permeable. For a sphere the optimal flows, in the sense of V. V. Pukhnachev, turned out to be potential flow. The ideal formulation of this problem is of interest. In this case the solution can be obtained simply, but the answer is nontrivial. In this connection there arises the following difficult question (which is not studied here): do close solutions exist in a fluid with low viscosity?

Let the sphere S with the radius 1 (all variables are dimensionless) be propelled from a state of rest in an ideal fluid, whose density is equal to 1, along the x-axis by the normal velocity of the fluid v_n (relative to S), which is a function of time t, given on S. Then the velocity potential of absolute motion φ satisfies the boundary condition

$$\left.\frac{\partial\varphi}{\partial n}\right|_S = v_n + \dot{x}_0\cos\theta = \sum_{k=0}^{\infty} c_k P_k(\cos\theta) + \dot{x}_0\cos\theta,$$

where $P_k(x)$ are Legendre polynomials and $P_1(x) = x$ (the flow is axisymmetric) (see Fig. 1); $\dot{x}_0$ is the velocity of the center of the sphere. We shall calculate the kinetic energy of the fluid

$$T_f = \frac{1}{2}\int\limits_{\text{outside } S} |\nabla\varphi|^2\,dx = \frac{1}{2}\sum_{k\neq 1}\frac{\alpha_k c_k}{k+1} + \frac{\alpha_1}{4}(c_1 + \dot{x}_0)^2$$
$$\left(\alpha_k = \int\limits_S P_k(\cos\theta)^2\,dS,\ \alpha_1 = \frac{4\pi}{3}\right). \quad (1)$$

Let P be the total momentum inside S. We transform the equation of motion into Lagrange's form and integrate once $P + \partial T_f/\partial\dot{x}_0 = \text{const} = 0.$ Substituting here the expression (1) we obtain

$$P + (2\pi/3)(\dot{x}_0 + c_1) = 0, \quad (2)$$

whence it is obvious that the regime is optimal for $c_k = 0$ $(k \neq 1)$.

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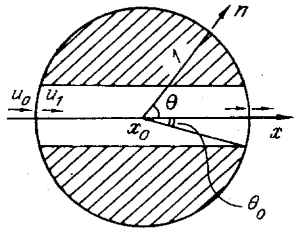

Fig. 1

We specify the form of v_n:

$$v_n = \begin{cases} u_0 \cos\theta & \text{at } 0 \leqslant \theta \leqslant \theta_0 \text{ and } \pi - \theta_0 \leqslant \theta \leqslant \pi, \\ 0 & \text{at } \theta_0 < \theta < \pi - \theta_0 \end{cases}$$

($0 < \theta_0 \leqslant \pi/2$ is a fixed parameter). An elementary calculation gives

$$c_1 = u_0(1 - \cos^3\theta_0), \tag{3}$$

and for $\theta_0 < \pi/2$ there will be other coefficients $c_k \neq 0$ ($k \neq 1$). Suppose that within S, in a cylinder whose radius is $\sin\theta_0$ and whose axis is also the x-axis, the fluid flows with constant velocity u_1 (relative to S) and the density of the fluid is ρ_1. If $\rho_1 \neq 1$, then on flowing into S through the front segment $\pi - \theta_0 \leq \theta \leq \pi$ the temperature of the fluid seemingly changes instantaneously and on flowing out through the back edge $0 \leq \theta \leq \theta_0$ the temperature jumps back to the starting value. The law of conservation of mass gives $\rho_1 u_1 = u_0$. We call attention to the fact that, generally speaking, the tangential component of the fluid velocity is discontinuous on the inlet and outlet segments. If $(4\pi/3)V$ is the volume of the cavity through which the fluid flows inside S and $(4\pi/3)m$ is the mass of the remaining solid part of the sphere, then

$$P = (4\pi/3)m\dot{x}_0 + (4\pi/3)V\rho_1(u_1 + \dot{x}_0). \tag{3'}$$

Substituting Eqs. (3) and (3') into Eq. (2) we obtain the formula sought:

$$\dot{x}_0 = -u_0(t)3V/[2(m + \rho_1 V) + 1] \ (V = 1 - \cos^3\theta_0).$$

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